Storage ring beam dynamics with two-frequency crab cavities

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NOCE 2017, Arcidosso, Italy,
September 20, 2017

U.S. DEPARTMENT OF ENERGY
Office of Science

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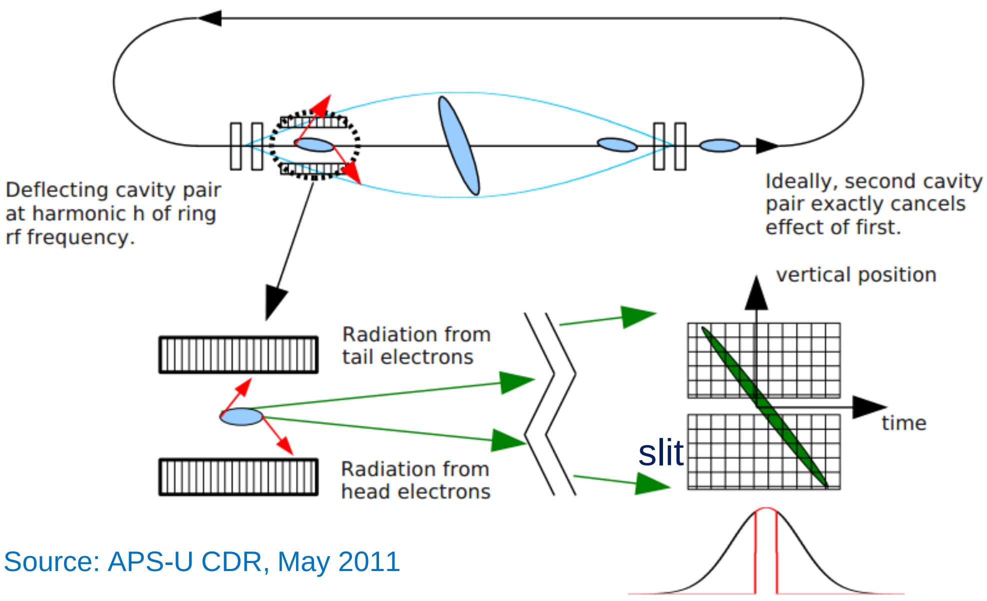
Outline

- Crab cavity for short pulse Concept
  - The new two-frequency crab cavity scheme
  - Components, system layout, and parameters
- Short pulse performance
  - What affect short pulse performance?
  - Two types of beamline optics
- Equilibrium distribution of tilted beam
  - Tilted distribution
  - Increase of vertical emittance
- Discussion of (selected) practical issues

“Crabbing” the beam for short pulses in storage rings

The tilt-and-cancel scheme

A. Zholents, P. Heimann, M. Zolotorev, and J. Byrd, NIM A 425, 385 (1999).

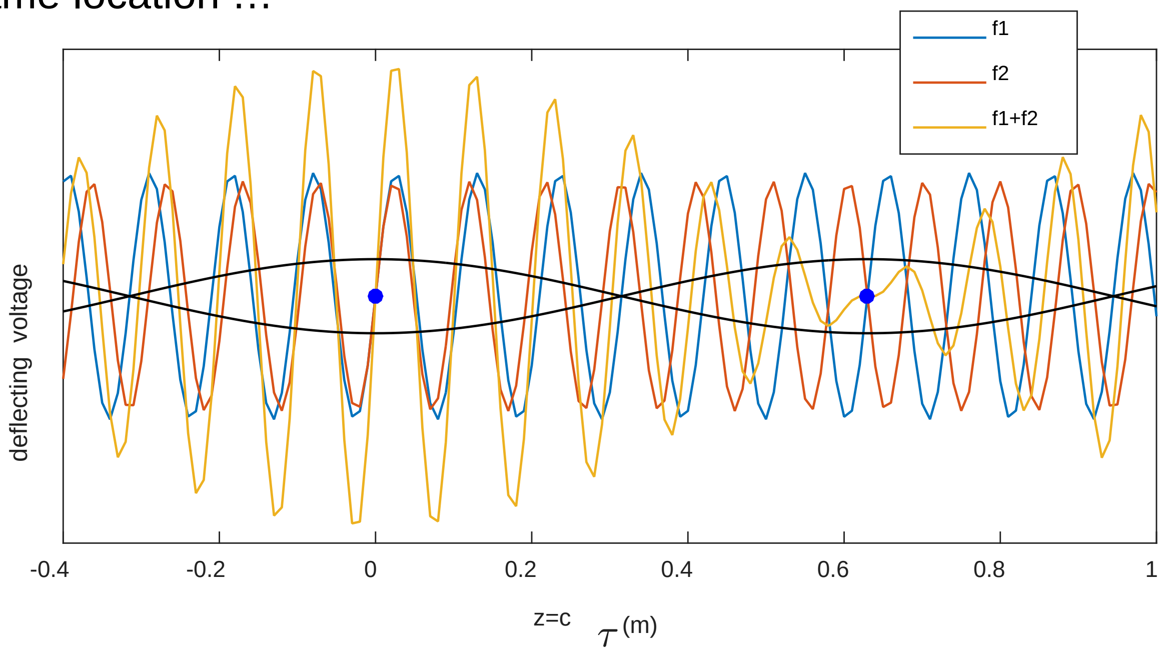


Source: APS-U CDR, May 2011

- The ID is $180° \times n_1$ downstream from crab cavity in vertical phase advance; radiation will have a maximum $y' - z$ correlation, which translates to $y - z$ correlation at a downstream slit.
- A second crab cavity is $180° \times n_2$ downstream from the first crab cavity to cancel the tilt.

The two-frequency crab cavity (2FCC) scheme*

If crab cavities of two frequencies are located at the same location …



Two frequencies:

$$f_1 = n f_0,$$

$$f_2 = \left(n + \frac{1}{2}\right) f_0$$

Half of the buckets are tilted in $y - z$ plane, the other half are un-affected.

To store regular, un-tilted beam, cancellation of crabbing kicks is needed. For the 2FCC scheme, cancellation occurs in time, not in space as the tilt-and-cancel scheme does.

*) Very similar to the BESSY VSR project (G.Wüstefeld et. al., IPAC 2011), but manipulate transverse tilts instead of the longitudinal focusing.

Advantages of the new scheme

- Short pulses are available all around the ring.
- No strict phase advance requirement for lattices.
- Crab cavities occupy only one straight section (and only one cryostat for SRF)
- Both cavities contribute to tilting and hence less total deflecting voltage is required.
- Beamlines can easily switch between short pulse mode and regular mode.
- Crab cavity can be used to separate short pulses from regular pulses.

Disadvantages:

- Crab cavities (and power source) of a second frequency are needed.
- Crab cavities add additional contribution to vertical emittance of the tilted bunch (to be discussed later), degrading X-ray flux and brightness.

Maximum deflecting voltage by minimum aperture

- The peak vertical kicks by crab cavities will deflect portions of the injected beam to the vacuum chamber.
  - The affected portion of the injected beam oscillates about the new closed orbit defined by the peak kick. So maximum offset is twice the kick

$$\frac{\sqrt{\beta_1\beta_2}}{2\sin\pi\nu_y}\frac{eV_d}{E}\times 2 = y_m$$

$y_m = 3.5$ mm the half aperture at $\beta_1 = 5$ m.

With $\beta_2 = 2.5$ m, $\nu_y = 6.32$, we found $V_d = 2.5$ MV (total deflecting voltage). Therefore, the maximum deflecting voltage for frequency 1 is
$V_1 = \mathbf{1.25}$ MV.

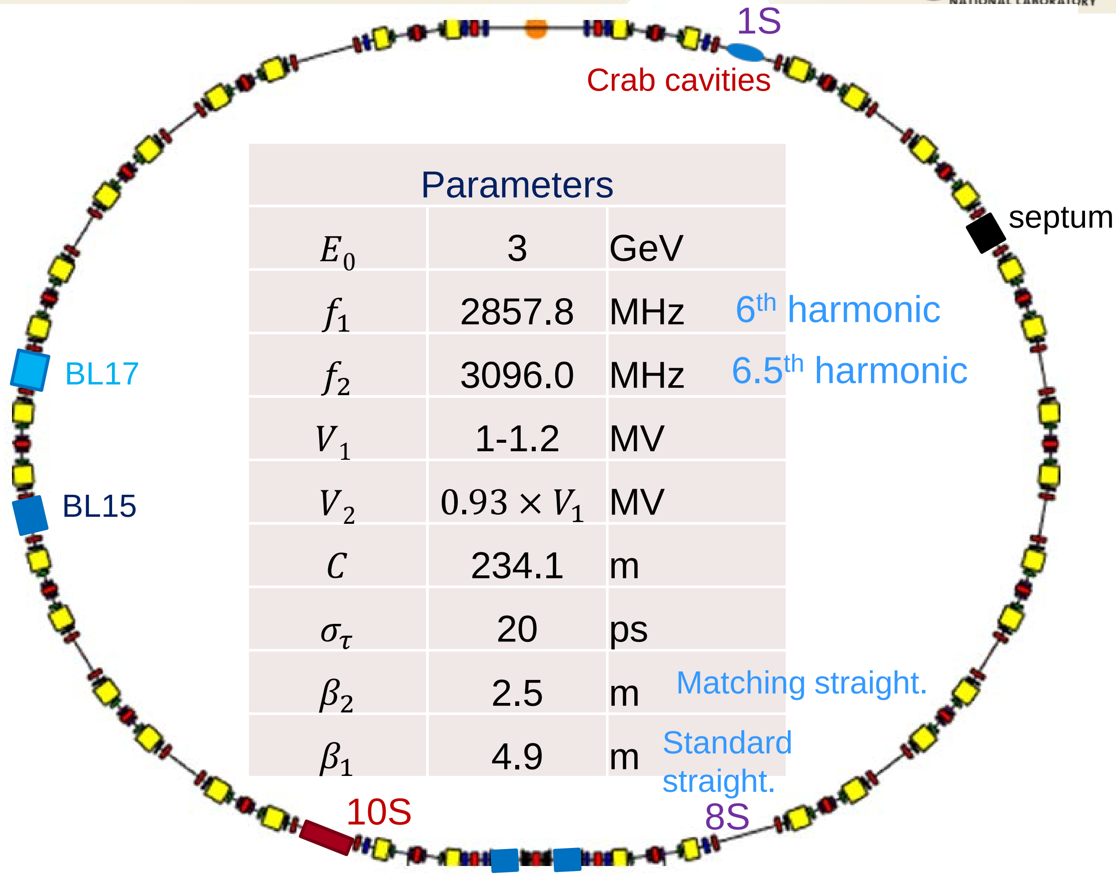
This is the limit by physical aperture and linear optics. Nonlinear motion may make things worse.

An example: application of 2FCC to SPEAR3

Crab cavity frequencies: 6^{th} and 6.5^{th} harmonics of 476.3 MHz.

Harmonic number is 372,
Revolution frequency 1.28 MHz.
Can have 1, 2, 4 camshaft bunches.

Fill Pattern:
480 mA in 140 regular buckets, and 20 mA to one camshaft bunch in the middle of a gap.



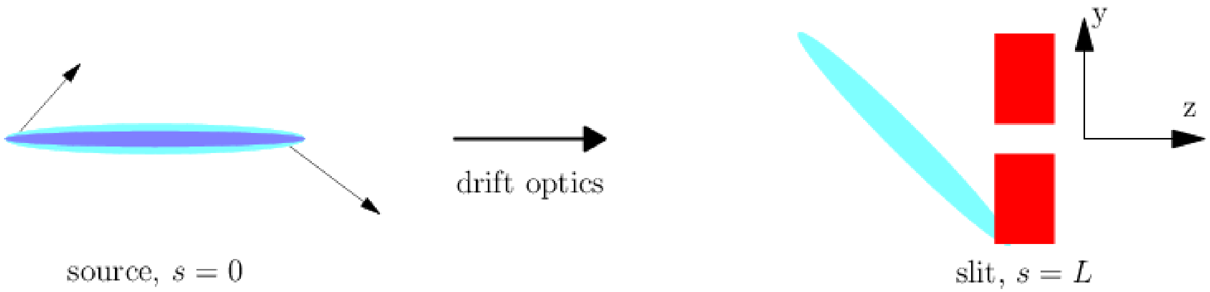
Parameters		
E_0	3	GeV
f_1	2857.8	MHz
f_2	3096.0	MHz
V_1	1-1.2	MV
V_2	$0.93 \times V_1$	MV
C	234.1	m
σ_τ	20	ps
β_2	2.5	m
β_1	4.9	m

Short pulse performance

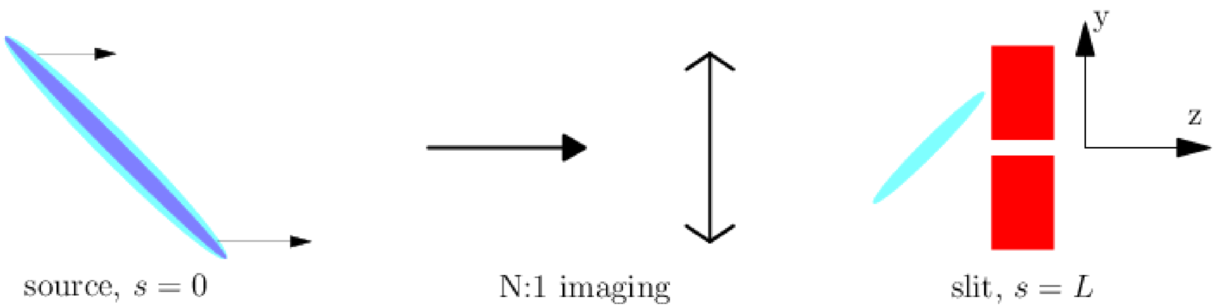
- Short pulse performance measures
  - Minimum photon pulse duration
  - Photon flux vs. pulse duration
- Two types of photon optics for short pulse selection
  - Drift and slit system
  - Imaging and slit system
  - There could be hybrid optics (not discussed here)
- What parameters affect short pulse performance?
  - Equilibrium distribution of tilted bunch
  - Lattice choice and impact
  - Crab cavity parameter choice

Two types of photon optics before short pulse selection

- There can be two types of beamline optics between source points and the short pulse selection slit:



Simple drift space: prefer large $y' - z$ slope at source.



In reality, mirror may introduce degradation to photon pulses.

Imaging optics: prefer large $y - z$ slope at source.

Photon beam distribution

- The photon distribution is related to the photon beam distribution at the source point via a transfer matrix (may also have degradation from mirror in case of imaging optics).
- The photon beam distribution at source point is the electron beam distribution convolved with single photon divergence and size

$\sigma_{y,ph}\sigma_{y',ph} = \frac{\lambda}{4\pi}$. (Undulator radiation assumed Gausian)

- The electron beam is at an equilibrium determined by the lattice and crab cavity parameters.
  - For a tilted beam, the projected beam sizes (σ_y, σ_z, σ_{yz}, etc) are related to the eigen emittances and the de-coupling transformation.

Formula for minimum pulse duration – drift optics

- For the drift-and-slit system, assuming at the source point the electron beam has a pure $y' - z$ slope (i.e., $C_{11} = 0$), and that the distance from SP to slit $L \gg \beta_y, \beta_{ph}$

A. Zholents, NIMA 798, 111 (2015)

$$\sigma_{zm} \approx \frac{1}{C_{21}} \sqrt{\frac{\epsilon_y}{\beta_y} + \sigma_\theta^2}$$

$C_{11} = \frac{dy}{dz}$, $C_{21} = \frac{dy\prime}{dz}$ are tilt slopes

This can be derived from

$$\sigma_y^2 = \left(L^2 + \beta_{ph}^2\right)\sigma_\theta^2 + \left(L^2 + \beta_y^2\right)\frac{\epsilon_y}{\beta_y} + (C_{11} + C_{21}L)^2\sigma_{z0}^2,$$
$$\sigma_{yz} = (C_{11} + C_{21}L)\sigma_{z0}^2,$$
$$\sigma_z \approx \sigma_{z0}.$$

where $\beta_{ph} = \frac{L_u}{2\pi}$, $\sigma_\theta = \sqrt{\frac{\lambda}{2L_u}}$ is photon divergence, ϵ_y is electron beam vertical slice emittance.

Hence the minimum photon pulse duration is

$$\sigma_{zm} = \frac{1}{C_{11} + C_{21}L} \sqrt{\left(\beta_y + \frac{L^2}{\beta_y}\right)\epsilon_y + (L^2 + \beta_{ph}^2)\sigma_\theta^2}$$

Formula for imaging optics

- With imaging optics, we slit the image of photon beam at source point. Ignoring the mirror induced degradation, the minimum pulse duration is

$$\sigma_{zm} \approx \frac{1}{C_{11}}\sqrt{\beta_y \epsilon_y + \sigma_{ph,y}^2}, \text{ with } \sigma_{ph,y} = \frac{\lambda}{4\pi\sigma_\theta}$$

It's desirable to maximize the y-z slope for the imaging optics case. Since for SPEAR3 $\beta_{ph} = \frac{L_u}{2\pi} \approx 0.5$ m is very small, for the imaging case, pulse duration is much LESS affected by diffraction than for the drift-and-slit case.

In any case, the equilibrium electron bunch distribution at source point is key to the short pulse performance. This requires the understanding of beam motion with crab cavity.

Crab cavity is a coupling device

E-M fields in a (vertical, assuming TM110) crab cavity (with $k = \frac{\omega}{c}$)

$$E_z = \mathcal{E}_0 k y \cos \omega t, \quad c B_x = \mathcal{E}_0 \sin \omega t,$$

Beam receives kicks (linearized) by a crab cavity

$$\Delta y' = \frac{eV}{E} k z, \quad \Delta \delta = \frac{eV}{E} k y,$$

The transfer matrix for a crab cavity for $\mathbf{X} = (y, y', z, \delta)^T$,

$$\mathbf{T}_c = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \epsilon & 0 \\ 0 & 0 & 1 & 0 \\ \epsilon & 0 & 0 & 1 \end{pmatrix}$$

with parameter $\epsilon \equiv \frac{eVk}{E}$

Crab cavity couples the longitudinal motion with a transverse plane (here vertical) much like skew quadrupole does for the horizontal-vertical planes.

Decoupling to normal modes

The coupled y-z motion can be decoupled to normal modes with a similarity transformation,

$$\mathbf{X} = \mathbf{V}\mathbf{X}_n$$

such that the new transfer matrix (for $\mathbf{X}_n$)

$$\mathbf{T}_{1n} = \mathbf{V}^{-1}\mathbf{T}_1\mathbf{V}$$

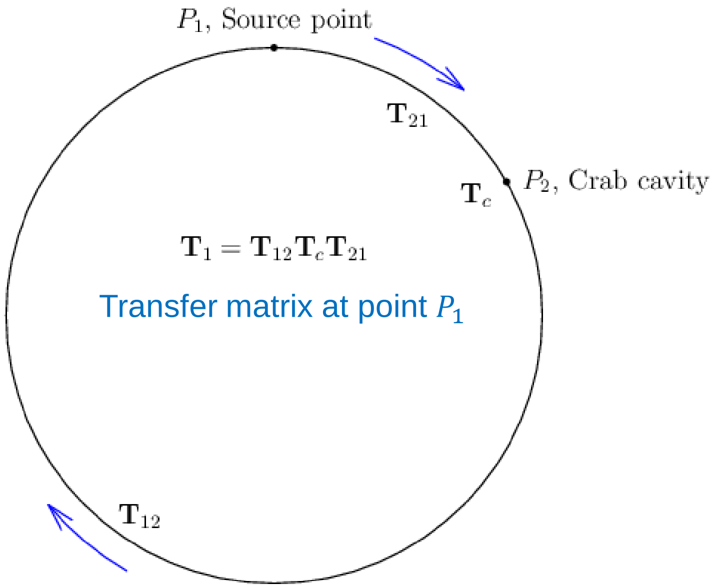
is block diagonal. The decoupling matrix can be written in the form

$$\mathbf{V} = \begin{pmatrix} r\mathbf{I} & \mathbf{C} \\ -\mathbf{C}^+ & r\mathbf{I} \end{pmatrix}$$

Where $\mathbf{C}^+$ is the symplectic conjugate of $\mathbf{C}$.

D. Edwards, L. Teng, IEEE Trans. Nucl. Sci. 20, 3 (1973)
D. Sagan, D. Rubin, PRSTAB 074001 (1999)

The transfer matrix with y-z coupling at any point can be calculated, which can then be block diagonalized.



Result of transfer matrix calculation

The original (w/o crab cavity) transfer matrix is

$$\mathbf{T}_1^{(0)} = \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{pmatrix}$$

and with crab cavity it becomes

$$\mathbf{T}_1 = \mathbf{T}_1^{(0)} + \epsilon \begin{pmatrix} \mathbf{0} & \tilde{\mathbf{T}}_{yz} \\ \tilde{\mathbf{T}}_{zy} & \mathbf{0} \end{pmatrix}$$

With $\tilde{\mathbf{T}}_{yz} = \mathbf{M}_{12}\mathbf{W}\mathbf{L}_{21}$ and $\tilde{\mathbf{T}}_{zy} = \mathbf{L}_{12}\mathbf{W}\mathbf{M}_{21}$

The decoupling matrix $\mathbf{C}$ is (to first order of ϵ)

$$\mathbf{C} = \frac{\epsilon(\tilde{\mathbf{T}}_{yz} + \tilde{\mathbf{T}}_{zy})}{\mathrm{Tr}(\mathbf{L} - \mathbf{M})}$$

Ignoring the synchrotron motion part, the results are

Slope in y-z: $\frac{dy}{dz} = C_{11}$

$$C_{11} = \epsilon \frac{\sqrt{\beta_1\beta_2}}{2\sin\pi\nu_y}\cos(\pi\nu_y - \Psi_{12}),$$

$$C_{12} = \epsilon \frac{\bar{\eta}\sqrt{\beta_1\beta_2}}{2\sin\pi\nu_y}\left[\frac{\sin\Psi_{12}}{2\sin\pi\nu_y} - \frac{\bar{\eta}_{12}}{\bar{\eta}}\cos(\pi\nu_y - \Psi_{12})\right],$$

Slope in y'-z: $\frac{dy'}{dz} = C_{21}$

$$C_{21} = \epsilon \frac{\sqrt{\beta_2/\beta_1}}{2\sin\pi\nu_y}[\sin(\pi\nu_y - \Psi_{12}) - \alpha_1\cos(\pi\nu_y - \Psi_{12})],$$

$$C_{22} = \epsilon \frac{\bar{\eta}\sqrt{\beta_2/\beta_1}}{2\sin\pi\nu_y}\left[\frac{1}{2\sin\pi\nu_y}(\cos\Psi_{12} - \alpha_1\sin\Psi_{12}) - \frac{\bar{\eta}_{12}}{\bar{\eta}}[\sin(\pi\nu_y - \Psi_{12}) - \alpha_1\cos(\pi\nu_y - \Psi_{12})]\right].$$

X. Huang, PRAB 19, 024001 (2016)

Tilted distribution

The equilibrium distribution at the source point is described by the Σ-matrix

$$\boldsymbol{\Sigma} \equiv \langle \mathbf{X}\mathbf{X}^T \rangle = \begin{pmatrix} \boldsymbol{\Sigma}_{\mathrm{yy}} & \boldsymbol{\Sigma}_{\mathrm{yz}} \\ \boldsymbol{\Sigma}_{\mathrm{yz}}^T & \boldsymbol{\Sigma}_{\mathrm{zz}} \end{pmatrix} = \mathbf{V}\boldsymbol{\Sigma}_{\mathrm{n}}\mathbf{V}^{\mathrm{T}},$$

with the de-coupled Σ-matrix being

$$\boldsymbol{\Sigma}_{\mathrm{n}} = \begin{pmatrix} \boldsymbol{\Sigma}_{\mathrm{y}} & \mathbf{0} \\ 0 & \boldsymbol{\Sigma}_{\mathrm{z}} \end{pmatrix}, \qquad \boldsymbol{\Sigma}_{\mathrm{y}} = \begin{pmatrix} \beta_y & -\alpha_y \\ -\alpha_y & \gamma_y \end{pmatrix} \epsilon_y, \text{ and } \boldsymbol{\Sigma}_{\mathrm{z}} = \begin{pmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_\delta^2 \end{pmatrix}$$

So, knowing the decoupling matrix $\mathbf{V}$, and the normal mode emittances, we can calculate the tilted equilibrium distribution anywhere in the ring, for example

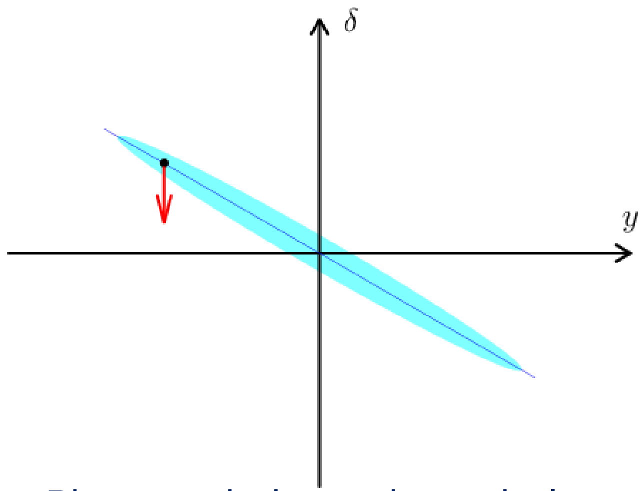
$$\boldsymbol{\Sigma}_{\mathrm{yz}} = \begin{pmatrix} \sigma_{\mathrm{yz}} & \sigma_{\mathrm{y}\delta} \\ \sigma_{\mathrm{y'z}} & \sigma_{\mathrm{y'}\delta} \end{pmatrix} \approx \mathbf{C}\boldsymbol{\Sigma}_{\mathrm{z}} = \begin{pmatrix} C_{11}\sigma_z^2 & C_{12}\sigma_\delta^2 \\ C_{21}\sigma_z^2 & C_{22}\sigma_\delta^2 \end{pmatrix}.$$

The eigen-emittances should not change under a symplectic transfer system. However, beam motion in electron storage ring is subject to radiation damping and quantum excitation and is thus not symplectic.

Through simulation we discovered an increase of vertical emittance due to tilted distribution in bending magnets.

Increase of vertical emittance due to crab cavities

The equilibrium beam distribution is tilted in y-δ and y'-δ directions



Photon emission excite vertical oscillation with y-z tilt

With emission of a photon of energy $\Delta\delta$, the electron's equilibrium position is shifted by

$$\Delta y = C_{12}\Delta\delta, \qquad \Delta y' = C_{22}\Delta\delta,$$

The betatron action variable is changed. The linear terms will average to zero. The quadratic terms will give rise an increase of emittance (balanced by radiation damping).

$$\Delta J_y = \mathcal{H}_c \Delta\delta^2, \qquad \mathcal{H}_c = \frac{1}{\beta_y}\left(C_{12}^2 + (\alpha_y C_{12} + \beta_y C_{22})^2\right).$$

Integrating over the entire ring, the emittance increase is found to be

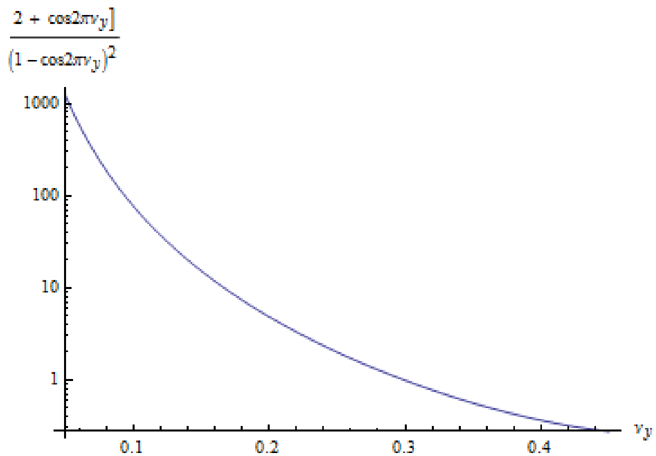
$$\epsilon_y = C_q \frac{\gamma^2 < \mathcal{H}_c >}{J_y \rho}, \qquad \langle \mathcal{H}_c \rangle = \frac{\epsilon^2 \alpha_c^2 C^2 \beta_2}{12} \frac{2 + \cos 2\pi\nu_y}{\left(\cos 2\pi\nu_s - \cos 2\pi\nu_y\right)^2}$$

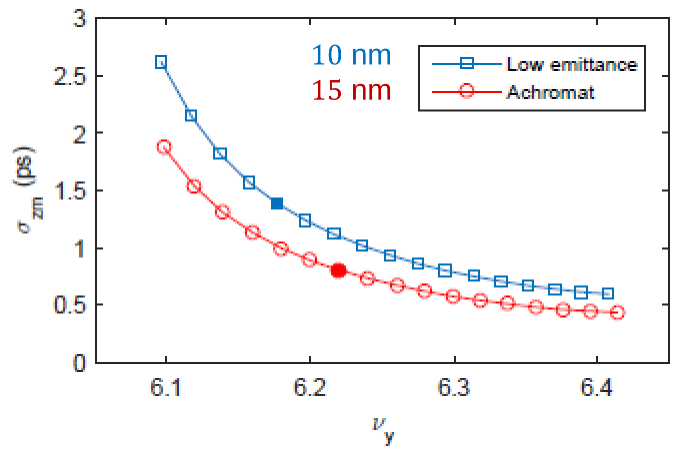
X. Huang, PRAB 19, 024001 (2016)

where α_c is momentum compaction factor and C is ring circumference.

Emittance dependence on vertical tune

The vertical eigen-emittance $\epsilon_y \propto \epsilon^2 \alpha_c^2$ and depends strongly on the vertical tune. Choosing ν_y toward half integer can significantly improve short pulse performance.





The vertical emittance contribution from tilted distribution may dominate. For example, for SPEAR3 with $\nu_y = 6.177$ and 2 MV deflecting voltage is: $\epsilon_{y1} = 680$ pm
As compared to vertical emittance from $x - y$ coupling $\epsilon_{y2} = 10$ pm

For this reason we choose a lattice with $\nu_y = 6.32$ for SPEAR3 (for a future lower emittance lattice), with $\epsilon_{y1} = 80$ pm .

Equilibrium distribution by Ohmi envelope calculation

- The equilibrium distribution (the $\mathbf{\Sigma}$-matrix) of the electron beam can be calculated by solving Ohmi's envelope equation

$$\mathbf{T\Sigma T}^{\mathrm{T}} + \overline{\mathbf{B}} = \mathbf{\Sigma}$$

K. Ohmi, K. Hirata, K. Oide, PRE 49, 1, 751 (1994)

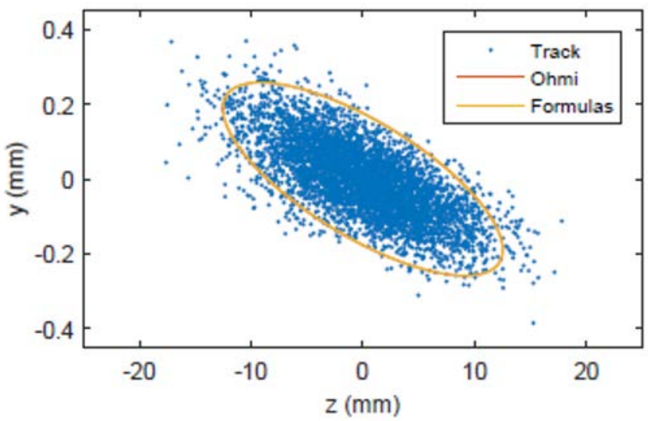
$\mathbf{T}$ is the one-turn transfer matrix, and $\overline{\mathbf{B}}$ is the one-turn integrated diffusion matrix, with

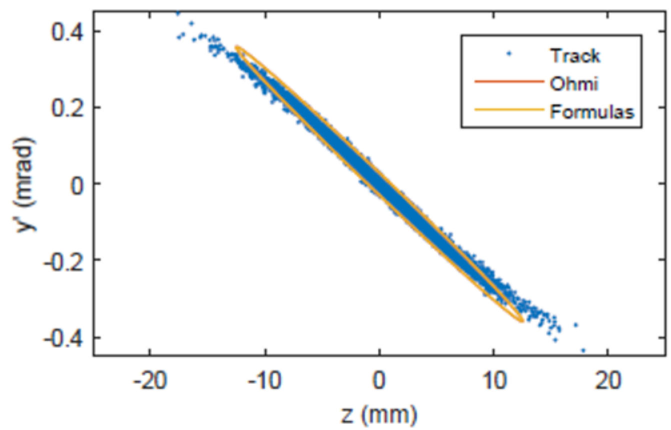
$$\overline{\mathbf{B}} = \int_{s}^{s+C} \mathbf{T}_{s+C,s'} \mathbf{B}(s')\, \mathbf{T}_{s+C,s'}^{T} ds'$$

and $\mathbf{B}(s')$ is the local diffusion matrix that represents the instantaneous impact to the distribution due to photon emissions.

Ohmi envelope calculation has been implemented in Elegant and AT.

Theory agrees with Ohmi envelope calculation and tracking simulation

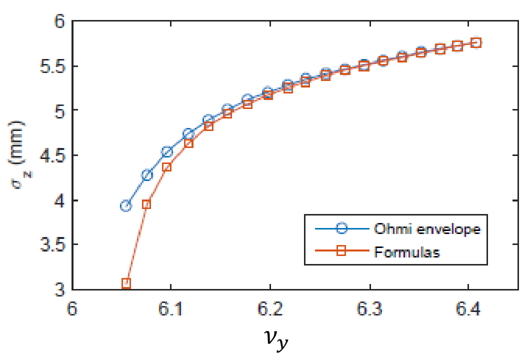


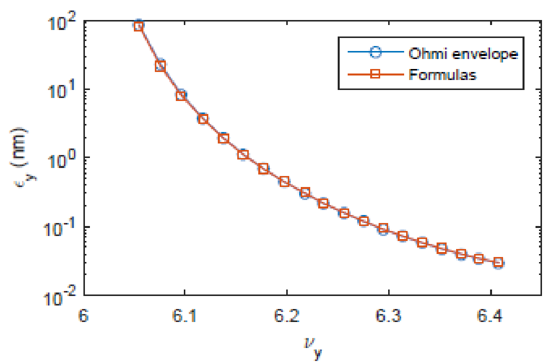


Formula overlaps with Ohmi envelope result.

Tracking was done with both AT and Elegant.

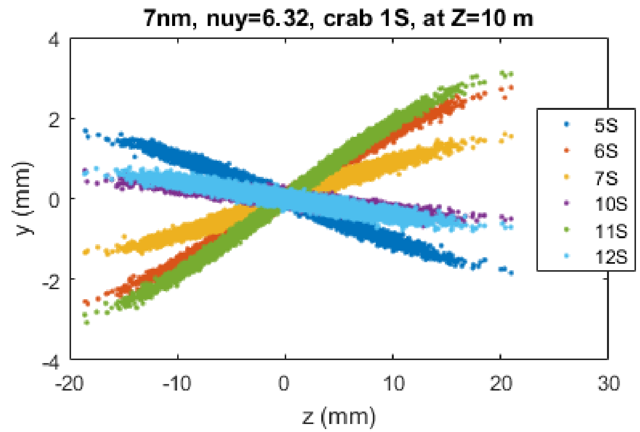
Equilibrium distribution from tracking vs. calculated ellipses for an example (6σ) for the 10nm lattice with $\nu_y = 6.177$.

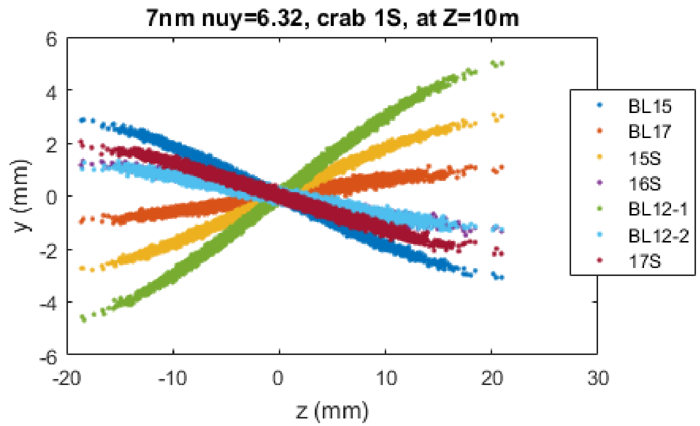


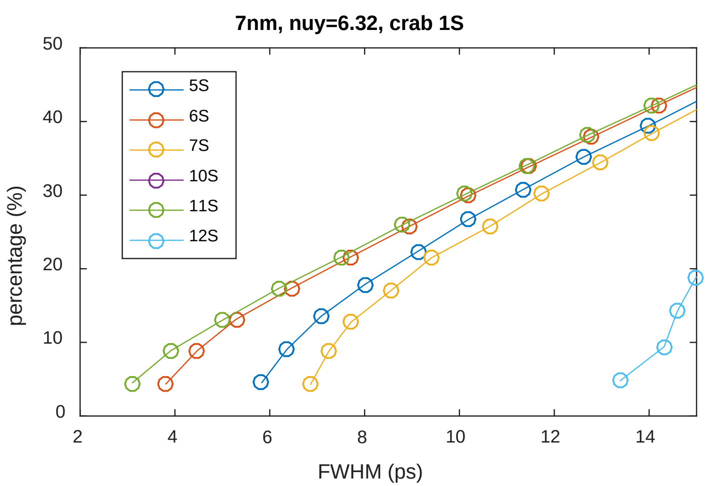


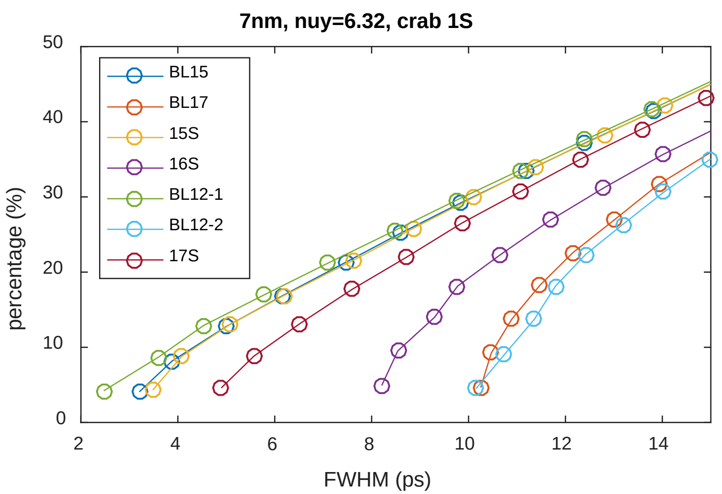
Crab cavity provides additional longitudinal focusing, which shortens the electron bunch.

Short pulse performance for SPEAR3 (w/ drift optics)









Many other beam dynamics aspects have been studied

- Impact to regular beam users
  - RF amplitude and phase noise
  - Phase shift by bunch train gap transient
- Injection into tilted buckets
- Beam lifetime (tilted, high current bunch)
- Crab cavity field requirements
- Separation of short pulse and long pulse
- Collective effects
  - Coupled bunch instabilities.
  - Single bunch current threshold (TMCI)
  - Microbunching instability.

We do not intend to cover these topics in this talk.

Summary

- The two-frequency crab cavity approach for short pulse generation in storage rings has many advantages over previous approaches.
- We have studied the coupled motion between the longitudinal and transverse directions due to crab cavities.
- Design study at SPEAR3 has made significant progress.

Acknowledgement

- Thanks to M. Borland for help in setting up Elegant tracking with crab cavities.
- Many input from SPEAR3 2FCC study team
  - Valery Dolgashev, Kelly Gaffney, Bob Hettel, Zenghai Li, Tom Rabedeau, James Safranek, Jim Sebek, Kai Tian.